

# PARALLEL COMPONENT $\mu_z$ OF PARTIALLY MAGNETIZED MICROWAVE FERRITES

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## ABSTRACT

A formula for the parallel component  $\mu_z$  of the microwave permeability tensor in the partially magnetized state is derived. The theory is in good agreement with experimental results.

## INTRODUCTION

In the actual applications of ferrites, microwave ferrites in the partially magnetized state are very important for the devices such as circulators operating below ferromagnetic resonance, latching phase shifters and so on. Therefore researches on the microwave permeability tensor of partially magnetized ferrites are obviously important. For a magnetic material magnetized in the z direction, the small-signal microwave permeability is a tensor of the form

$$\vec{\mu} = \begin{pmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \quad (1)$$

The parallel component  $\mu_z$  is not equal to unity for partially magnetized materials. Rado [1] obtained the off-diagonal element  $k$  in nonsaturated ferromagnetic materials under some conditions and assumptions. The parallel element  $\mu_z$  from Rado's theory did not agree with the experimental results. Later, Schloemann [2], [3] presented a formula for the (isotropic) permeability in the completely demagnetized state. When there exists an external dc magnetic field of arbitrary strength, Schloemann's formula cannot be used. The authors [4] presented formulas for the complex transverse components of partially magnetized ferrites derived under some assumptions. The theoretical formulas can represent the experimental values for any strength of the externally applied dc magnetic field with good agreement. Only a few reports are, however, on the parallel component  $\mu_z$  of partially magnetized ferrites, for example [5], [6] and [7]. As far as the authors know there has been only an empirical formula for the parallel component  $\mu_z$  of partially magnetized ferrites; Green et al. [6], [7] proposed that an empirical equation for the real part of the parallel component  $\mu'_z$  be given by

$$\mu'_z = \mu'_0 [1 - (M/M_s)^{5/2}] \quad (2)$$

where  $\mu'_0$  is the real part of the permeability in the completely demagnetized state [2], [3],  $M$  the magnetization,  $M_s$  the saturation magnetization.

There are many microwave ferrite devices which operate in the region below saturation. In this region, the real part of the parallel component  $\mu'_z$  is, as shown later, less than unity, particularly for the completely demagnetized state. The imaginary part  $\mu''_z$  has a large value for

$$\frac{\omega_M}{\omega} \geq \frac{5}{3}$$

[4]. Thus, the attenuation due to  $\mu''_z$  can not be neglected. Therefore, it is obviously important to formulate the complex parallel component  $\mu_z$ .

In this paper, using averaging techniques, we first present a formula for the complex parallel component of the microwave permeability tensor in the partially magnetized state. Lastly, the theory will be compared with the experimental results.

## FORMULATION FOR $\mu_z$

If the magnetic inhomogeneities are larger than the domain wall thickness, the exchange torque may be neglected at points which are not contained within walls. Consequently, at such points, the magnetization vector  $\vec{M}(t)$  and effective magnetic field vector  $\vec{H}_e(t)$  are related by Gilbert's equation [8]; where  $\gamma$  is the gyromagnetic ratio and  $\alpha$  the phenomenological loss term.

Furthermore a time-independent unit vector  $\vec{u}$  shows the equilibrium direction of the saturation magnetization vector  $\vec{M}_s$  at the point in question. Since the magnetization within the domain is saturated, the unit vec-

$$\frac{d\vec{M}(t)}{dt} = \gamma \vec{M}(t) \times \vec{H}_e(t) + \frac{\alpha}{M_s} \vec{M}(t) \times \frac{d\vec{M}(t)}{dt} \quad (3)$$

tor  $\vec{u}$  is defined as follows;

$$\vec{M}(t) = M_s \vec{u} + \vec{m}(t) \quad (4)$$

where  $\vec{m}(t)$  is the RF magnetization vector. Writing down the RF magnetic field  $\vec{h}(t)$ , the locally effective magnetic field  $\vec{H}_e(t)$  is written as follows;

$$\vec{H}_e(t) = H_e \vec{u} + \vec{h}(t) \quad (5)$$

where  $H_e \vec{u}$  is a locally effective magnetostatic field and represents the u-component of the resultant field of the applied field, anisotropy field and so on.

In small signal analysis, assume that  $m/M_s \ll 1$  and  $h/H_e \ll 1$ . Calculating a vector product of Eq.(3) and  $\vec{u}$ , and arranging Eq.(3) by substituting

$$\vec{m} \times \vec{u}$$

obtained here into Eq.(3), we obtain, with the time-dependence of  $\exp(j\omega t)$

$$\vec{m} = -\left(\frac{\omega}{j\omega} + \alpha\right)\vec{m} + \frac{\mu_0 \omega_M}{j\omega} \left(\frac{\omega_e}{j\omega} + \alpha\right) \left\{ \vec{h} - (\vec{h} \cdot \vec{u})\vec{u} \right\} + \frac{\mu_0 \omega_M}{j\omega} \vec{h} \times \vec{u} \quad (6)$$

where  $\omega_e = -\gamma H_e$ ,  $\omega_M = -\gamma M_s / \mu_0$

and  $\mu_0$  = the intrinsic permeability of free space. Letting  $\vec{u} = i\alpha_1 + j\alpha_2 + k\alpha_3$ ,

where  $i, j, k$  are unit vectors along the x, y, z coordinate axes, respectively, for the case where the magnetic material is magnetized parallel to the z-axis the parallel component  $\mu_z$  is given as

$$\mu_z = 1 + \frac{\omega_M(\omega_e + j\omega\alpha)}{-\omega^2 + (\omega_e + j\omega\alpha)^2} (\alpha_1^2 + \alpha_2^2) \quad (7)$$

The direction of the domain may be sufficiently random, particularly in a polycrystal, that is,

$$\langle \alpha_1 \rangle = 0, \langle \alpha_2 \rangle = 0, \langle \alpha_3 \rangle = M/M_s$$

$$\langle \alpha_1^2 \rangle = \langle \alpha_2^2 \rangle = \frac{1}{2}(1 - \langle \alpha_3^2 \rangle) \quad (8)$$

where symbol  $\langle \rangle$  means a spatial average and M is an average magnetization of the z-direction. Consequently, for partially magnetized materials with random domain orientation, the parallel component of the effective permeability tensor of two or more domains  $\langle \mu_z \rangle$  is given by

$$\langle \mu_z \rangle = 1 + \frac{\omega_M(\omega_e + j\omega\alpha)}{-\omega^2 + (\omega_e + j\omega\alpha)^2} (1 - \langle \alpha_3^2 \rangle) \quad (9)$$

The parallel component  $\mu_z$  from Rado's theory is independent of an average magnetization M but is equal to unity. Consequently, Rado's  $\mu_z$  did not agree with the experimental results [5], [6], [7]. The parallel component  $\langle \mu_z \rangle$  in Eq.(9) is a quadratic function of  $\alpha_3$ . As we increase M, and the magnetization approaches saturation in the z direction  $\langle \alpha_3^2 \rangle \rightarrow 1$ , the real part of  $\langle \mu_z \rangle$  approach-

es 1 but on the other hand the imaginary part of  $\langle \mu_z \rangle$  approaches 0. This is physically very reasonable. Eq.(9) is not rigorous because we cannot theoretically calculate  $\omega_e$ . As shown in the next section, however, Eq.(9) does hold rigorously if  $\omega_e$  is obtained from experiment.

#### COMPARISON OF THE EXPERIMENTAL RESULTS

In Eq.(9) we express  $1 - \langle \alpha_3^2 \rangle$  by approximating it as follows;

$$1 - \langle \alpha_3^2 \rangle \simeq 1 - \langle \alpha_3 \rangle^2 = 1 - (M/M_s)^2 \quad (10)$$

Substituting Eq.(10) into Eq.(9), the real part and the imaginary part of parallel component  $\langle \mu_z \rangle$  are  $\mu'_z$  and  $\mu''_z$ , respectively.

The formula for the parallel component has been compared to measurements by Green et al.

[6] of  $\mu'_z$  and  $\mu''_z$  as a function of  $M/M_s$ .

In fitting the theoretical formula to the experimental results the parameters  $\omega_e$  and  $\alpha$  have to be suitably chosen, since they cannot be independently determined. Typical results are shown in figures 1 - 3. Comparison of the experimental results and the theory in figures 1 - 3 shows that the agreement is reasonably good throughout the partially magnetized region ( $-1 \leq M/M_s \leq 1$ ). The value of imaginary part differs slightly from the experimental value, as  $M/M_s$  approaches 1. The value of theoretical formula  $\mu_z$  becomes equal to 1 (real number) at  $\langle \alpha_3^2 \rangle = 1$ . This makes little difference between the theory and the experimental results because the imaginary part of  $\mu_z$  has actually a very small value even if the magnetization became saturation in the z direction.

#### CONCLUSION

A formula for the parallel component of the microwave permeability tensor of partially magnetized materials is for the first time derived as a function of magnetization. The theory is useful for partially magnetized devices and is in good agreement with the experimental results.

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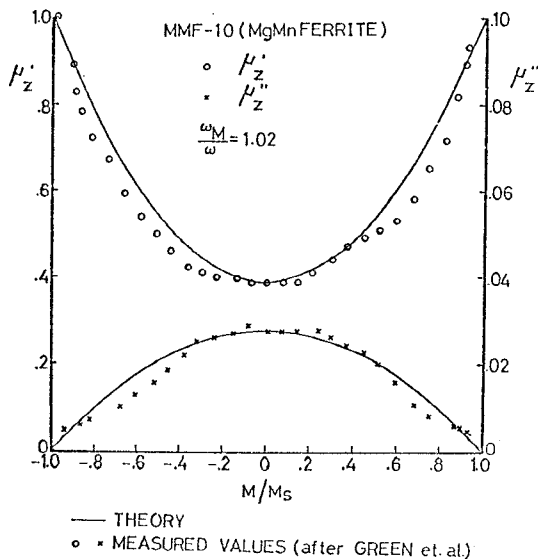


Fig. 1 Real and imaginary parts of parallel component of partially magnetized ferrites as a function of  $M/M_s$ .  
 $\omega_e = 16.2$  GHz,  $\alpha = 1.35 \times 10^{-2}$ ,  
 $f = 5.5$  GHz. (Green et al. [6]).

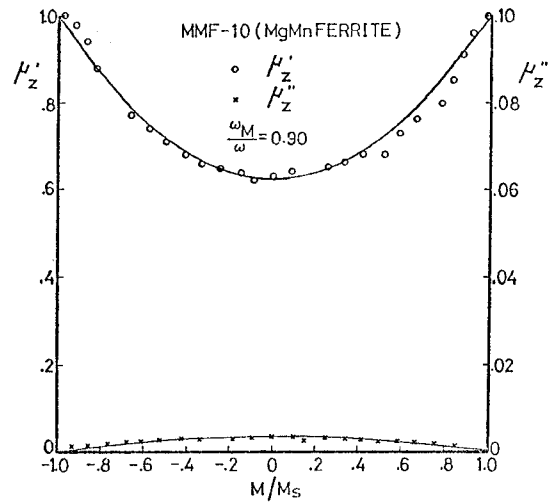


Fig. 2 Real and imaginary parts of parallel component of partially magnetized ferrites as a function of  $M/M_s$ .  
 $\omega_e = 12.5$  GHz,  $\alpha = 0.25 \times 10^{-2}$ ,  
 $f = 5.5$  GHz. (Green et al. [6]).

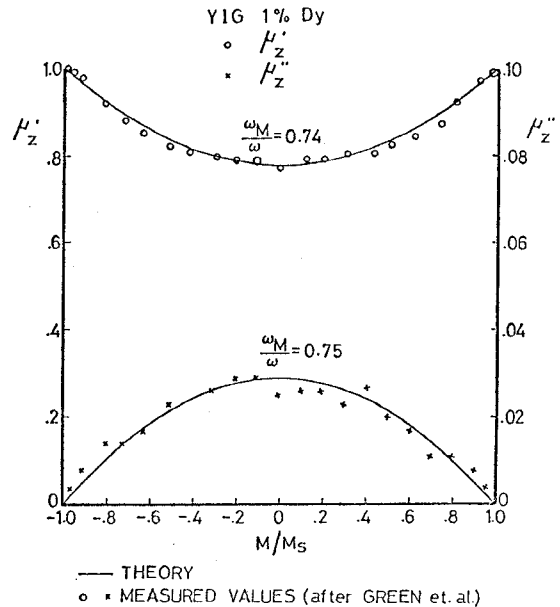


Fig. 3 Real and imaginary parts of parallel component of partially magnetized ferrites as a function of  $M/M_s$ .  
 $\omega_M/\omega = 0.74$ ;  $\omega_e = 9.5$  GHz,  $\alpha = 0.29 \times 10^{-2}$   
 $\omega_M/\omega = 0.75$ ;  $\omega_e = 9.6$  GHz,  $\alpha = 0.30 \times 10^{-2}$   
 $f = 5.5$  GHz. (Green et al. [6]).